



Sales Maximisation and Tacit Collusion under Multimarket Contact: A Model-Based Analysis

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Abstract: This study examines how the conditions supporting tacit collusion across multiple markets change when firms maximise sales revenue rather than profit. An infinite-horizon repeated-game model compares profit and sales-revenue objectives in single- and two-market settings, with sales maximisation subject to a non-negative-profit constraint. The analysis shows that market share and market growth influence cooperative stability under both objectives, but sales maximisation introduces an additional determinant: the proportional change in aggregate market sales following a price war. When punishment preserves substantial sales, the future loss from deviation is smaller and a higher discount factor is required to sustain cooperation. Under multimarket contact, the relevant market-share term becomes the average share across linked markets, so contact can stabilise cooperation when asymmetries across markets offset one another, but it does not automatically strengthen collusion in symmetric settings. Under linear demand, the demand-response parameter is determined by demand primitives, and the sales-maximisation threshold equals the profit-maximisation threshold multiplied by the price elasticity of demand at the collusive price. The present paper develops the earlier sales-maximisation framework by deriving this linear-demand characterisation and by positioning the resulting conditions within recent research on managerial incentives and multimarket contact.

Key Words : tacit collusion; multimarket contact; sales maximisation; managerial objectives; repeated games; mutual forbearance.

1. INTRODUCTION

Firms often meet the same rivals in more than one market. Airlines compete on many routes, banks operate across many local areas, and diversified manufacturers encounter one another in several product categories. In these settings, an action in one market can affect expectations and responses elsewhere. A firm that cuts price to gain share in one market may face retaliation in another market where it is more vulnerable. The possibility of cross-market retaliation can restrain aggressive behaviour and produce what the literature calls mutual forbearance. Theoretical and empirical work has consequently associated multimarket contact with higher or more stable prices, fewer aggressive competitive moves and, in some settings, improved firm performance (Bernheim and Whinston 1990; Evans and Kessides 1994; Gimeno and Woo 1996; Karnani and Wernerfelt 1985; Li and Greenwood 2004; Young et al. 2000).

The standard explanation is built on profit maximisation. Firms compare the short-run profit from deviation with the discounted loss of future collusive profit. Contact across several markets can pool these incentive constraints: a weak incentive to cooperate in one market may be offset by a stronger incentive in another. This logic is elegant and influential, but the objective assigned to the firm is not a neutral modelling choice. Managerial theories of the firm argue that managers can value sales, growth, organisational size and employment as well as profit, particularly when ownership is dispersed or when stakeholder commitments constrain restructuring (Baumol 1959; Marris 1964; Williamson 1964). Comparative research also shows that the weight placed on shareholders, employees and continuity differs across corporate systems (Dore 2000; Yoshimori 1995).

These differences matter for punishment. In a conventional repeated Bertrand model, a deviation provokes price competition and eliminates supracompetitive profit. Future profit during punishment is therefore normalised to zero. A sales-maximising firm, however, continues to record sales after prices fall. If lower prices expand demand sufficiently, aggregate sales revenue during the punishment phase may remain high or may even exceed revenue under the cooperative price. A price war that is severe from a profit perspective may consequently be much less severe from a sales perspective. The standard punishment mechanism can therefore lose deterrent power when the corporate objective changes.

This paper asks: under what conditions can tacit collusion be sustained when firms maximise sales revenue rather than profit, and how does multimarket contact alter those conditions? The paper develops an infinite-horizon repeated-game framework for two firms that produce a homogeneous product and compete in either one market or two linked markets. The single- and two-market sales-maximisation conditions build directly on the author's earlier Japanese-language analysis (Sakai 2017), which introduced the sales objective and the punishment-response parameter. The present paper restates that framework transparently for an international readership, updates its theoretical positioning, and adds a closed-form characterisation of the demand-response parameter under linear demand.

The analysis produces four main results. The first three restate and sharpen the earlier framework in Sakai (2017): the sales-maximisation condition depends on the change in aggregate market sales during punishment; sustaining tacit collusion generally requires greater future orientation under sales maximisation than under profit maximisation; and multimarket contact affects the incentive constraint through the average or weighted-average market position across linked markets rather than through overlap alone. The fourth result is new to the present paper: under linear demand, the demand-response parameter is determined by demand primitives, and the sales-maximisation threshold equals the profit-maximisation threshold multiplied by the price elasticity of demand at the collusive price—equivalently, divided by the Lerner index. This provides a direct economic interpretation of the punishment response in terms of an observable mark-up.

Relative to Sakai (2017), the main theoretical contribution of the present paper is the linear-demand characterisation of the punishment response and its integration with recent work on managerial incentives, decentralised decision making and non-price outcomes. The practical implication is that managers and competition authorities should not infer stable restraint from market overlap alone. Where sales growth, employment or scale is strongly valued, and where price reductions stimulate demand, aggressive competition may remain attractive despite extensive multimarket contact.

The remainder of the paper proceeds as follows. Section 2 reviews research on multimarket contact and managerial objectives and identifies the unresolved link between them. Section 3 sets out the analytical method and derives the profit- and sales-maximisation conditions. Section 4 compares the results and discusses their implications. Section 5 concludes and outlines limitations and directions for further research.

2. REVIEW OF LITERATURE

2.1 Multimarket Contact and Mutual Forbearance

The concept of multiple-point competition emphasises that the relevant unit of rivalry is not always a single product or geographic market. When two firms encounter each other repeatedly across a portfolio of markets, actions can be linked across those encounters (Karnani and Wernerfelt 1985). Bernheim and Whinston (1990) formalised this intuition in repeated games. Their central result is conditional rather than automatic: multimarket contact can facilitate collusion when firms or markets are asymmetric, because linking markets allows otherwise binding incentive constraints to be pooled. If markets and firms are perfectly symmetric and independent, simply replicating the same interaction need not enlarge the set of sustainable outcomes.

The strategic-management literature translates this formal result into mutual forbearance. A firm refrains from attacking a rival because the rival can respond in several shared markets. The threat is particularly credible when each firm possesses a favourable position in at least one market and a vulnerable position in another. Each recognises that a local gain can trigger a broader loss. Multimarket rivalry can also improve information about a competitor's intentions and capabilities because repeated encounters create more opportunities to observe behaviour (Gimeno and Woo 1996; Yu and Cannella 2013).

Empirical findings broadly support the relevance of this mechanism while also showing that its effect depends on context. Evans and Kessides (1994) found that extensive inter-route contact in the United States airline industry was associated with higher fares. Gimeno and Woo (1996) showed that strategic similarity and multimarket contact contributed to competitive de-escalation among airlines. Young et al. (2000) found that software firms with more multimarket contact initiated fewer competitive actions, although their response timing also depended on resource similarity. Li and Greenwood (2004) demonstrated that multimarket contact interacted with market structure in the Canadian insurance industry. More recent structural evidence indicates that the degree to which contact facilitates tacit coordination depends on demand substitution and conduct within the industry (Ciliberto and Williams 2014).

The effects are not limited to price. Wilson (2025) found that greater multimarket contact among internet service providers was associated with lower download speeds, consistent with mutual forbearance operating through product quality. This matters because cooperation may appear as reduced investment, slower innovation or fewer competitive actions rather than an observable price agreement. Feinberg and Larson (2024) further showed theoretically that multimarket contact may support tacit collusion even when market-level managers make decentralised decisions and care only about profit in their own market. Their informational mechanism demonstrates that the internal structure of the firm changes how contacts are linked.

Recent work continues to refine when contact matters and through which channels. Benischke and Bhaskarabhatla (2026) study how multimarket contact affects pricing strategies in markets under regulatory threat, and research on algorithmic pricing shows that learning algorithms can sustain supracompetitive prices through reward-and-punishment schemes without explicit communication (Calvano et al. 2020). In both strands, as in the classical models, the objective that pricing decisions maximise is profit; whether the same mechanisms operate when the decision maker values a different outcome remains open.

Taken together, the literature establishes three points. First, rivalry can be linked across markets. Second, the effect of that linkage is contingent on asymmetry, monitoring, market structure and organisational arrangements. Third, most formal and empirical interpretations evaluate outcomes through profit. The comprehensive review by Yu and Cannella (2013) documents extensive research on antecedents, outcomes and moderators of multimarket contact, but alternative managerial objectives remain comparatively underdeveloped. The present paper focuses on that omission.

2.2 Profit Maximisation as the Standard Premise

In the standard repeated-game account, firms maintain a cooperative price because the present value of future supracompetitive profit exceeds the one-period gain from deviation. If a firm deviates, rivals implement a punishment strategy, commonly modelled as a reversion to Bertrand competition. With homogeneous products and identical marginal costs, the punishment price equals marginal cost and economic profit falls to zero. The threat

is effective because deviation sacrifices an indefinite stream of future profit.

Multimarket contact can reinforce this threat by allowing punishment in more than the market where deviation occurred. The deviator must compare the local gain with losses across the full set of shared markets. When a firm has a small share in one market but a large share in another, the second market raises the opportunity cost of destabilising cooperation. The relevant incentive then depends on its combined position across markets. This pooling logic underlies much of the mutual-forbearance hypothesis (Bernheim and Whinston 1990).

The profit objective is reasonable for many applications. Profit finances distributions to investors and investment in future operations; it also provides a common metric across markets. Nevertheless, assuming profit maximisation excludes variation in governance, managerial incentives and stakeholder commitments by construction. It cannot answer whether the same punishment remains effective when the firm values a different outcome.

2.3 Managerial Objectives and Sales Maximisation

Managerial theories of the firm question whether managers always act as pure profit maximisers. Baumol (1959) proposed sales-revenue maximisation subject to a minimum-profit constraint. Sales can matter because managerial remuneration, organisational prestige, bargaining power and access to resources are often related to the scale of the enterprise. Marris (1964) similarly connected managerial utility to corporate growth, while Williamson (1964) analysed discretionary behaviour made possible by separation between ownership and control. These theories do not claim that profit is irrelevant. Rather, they suggest that once a feasible profit requirement has been met, managers may prefer a strategy that produces greater sales or growth.

Stakeholder orientation provides a second route to a sales objective. A firm that places substantial weight on stable employment may resist capacity reduction or market exit because those actions impose costs on employees. Maintaining or expanding sales can help utilise capacity and preserve jobs even when margins are thin. Comparative evidence has long identified variation in corporate purpose. Yoshimori (1995) reported marked differences between Japanese and Western executives in whether the corporation was understood primarily as belonging to shareholders or to a broader set of stakeholders. Dore (2000) likewise contrasted shareholder-oriented capitalism with systems in which employees, suppliers and organisational continuity receive greater weight.

A related industrial-organisation literature analyses how managerial incentives shape oligopoly conduct. In the strategic-delegation tradition, owners can reward managers on a combination of profit and sales, which alters the intensity of product-market competition (Fershtman and Judd 1987). In repeated interaction, Spagnolo (2005) shows that the structure of managerial incentive schemes affects the sustainability of collusion, and Lee and Turner (2024) show that sales-based compensation, which does not facilitate collusion between identical firms, can facilitate collusion when firms differ in costs or product quality. Empirically, Antón et al. (2023) relate common ownership to less performance-sensitive

managerial incentives and to product-market outcomes, while Ha et al. (2024) find that firms adjusted executive pay towards rivals' performance when antitrust enforcement weakened. These studies confirm that the objective guiding pricing decisions is an economically important variable. They focus, however, on single-market interaction or on the design of incentive contracts; they do not examine how a sales objective interacts with the pooling of incentive constraints across linked markets or with the demand response to punishment, which is the focus of the present paper.

These arguments imply that punishment based on lost profit may not translate directly into punishment based on lost sales. Under Bertrand punishment, positive economic profit can disappear while transactions continue. If the price reduction raises quantity demanded enough to preserve aggregate revenue, a sales-oriented manager may regard the punishment phase as tolerable. If it expands aggregate revenue, the punishment phase can even be attractive on the objective that the manager is assumed to maximise. In contrast, when price reductions fail to expand quantity, aggregate revenue can contract sharply and the punishment remains powerful.

The distinction is especially relevant across product-life-cycle stages and demand regimes. During diffusion, a high price can exclude price-sensitive consumers, so a price reduction may enlarge the customer base. In a mature or declining market, the same reduction may redistribute customers between suppliers without materially increasing total demand. The growth of the market over time and the immediate revenue response to a price war are conceptually different. A growing market raises future cooperative sales, whereas a price-war response changes the relative attractiveness of the punishment path. A model that includes only the growth rate cannot separate these two mechanisms.

2.4 Research Gap and Analytical Propositions

The literature therefore contains two developed but insufficiently connected arguments. Multimarket-contact theory explains how linked punishment can restrain rivalry, while managerial theories explain why firms may pursue sales, growth or stakeholder continuity rather than profit alone. The unresolved question is whether linked punishment works in the same way under a sales objective.

The analysis below evaluates five propositions. First, under profit maximisation, a larger market share and a higher market growth rate lower the degree of future orientation required to sustain tacit collusion. Second, under sales maximisation, these effects remain, but the threshold also depends on the change in aggregate market sales during punishment. Third, when the market continues to generate sales during punishment, the sales-maximisation threshold is more demanding than the profit-maximisation threshold. Fourth, in two linked markets, the relevant share is the average share across those markets; multimarket contact therefore facilitates restraint when the linkage relaxes an otherwise binding constraint, not merely because two markets exist. Fifth, under linear demand, the change in aggregate sales during punishment is determined by demand primitives, so that the sales-maximisation threshold can be expressed through the price elasticity of demand at the collusive price.

Relationship to prior work. The present paper builds on Sakai (2017), a Japanese-language article published in a university bulletin, which introduced a sales-maximisation objective into the multimarket-contact model, derived the single- and two-market conditions restated in Sections 3.2–3.4, and analysed a mixed model in which a sales-maximising firm faces a profit-maximising rival. The present paper therefore does not claim those baseline conditions as new. Its principal new analytical result is Proposition 1 in Section 3.5, which derives the demand-response parameter from a linear demand system and shows that the sales-maximisation threshold equals the corresponding profit threshold multiplied by the price elasticity of demand at the collusive price. In addition, the paper makes the parameter restrictions and boundary cases explicit, separates market growth from the immediate demand response to punishment, updates the literature on managerial incentives and multimarket contact, and develops managerial and competition-policy implications. The mixed-objective results are summarised in Section 4.5 with reference to Sakai (2017) rather than re-derived.

3. RESEARCH METHODOLOGY

3.1 Analytical Approach and Assumptions

The study uses a theoretical model rather than an empirical dataset. It compares discounted streams of profit or sales under two strategies: continuing tacit collusion and deviating once, followed by an indefinite punishment phase. This approach isolates the effect of the objective function while holding the basic competitive environment constant. No proprietary software, questionnaire, scale or licensed dataset is used, and no permission is required to replicate the analysis.

Two firms, Firm 1 and Firm 2, produce a homogeneous product with identical constant marginal costs. They interact for an infinite number of periods. The discount factor is denoted by d , where $0 < d < 1$; a larger value means that future outcomes receive greater weight. The market grows at the exogenous rate g , so the scale of profit or sales in period t is multiplied by $(1 + g)^t$, with $g > -1$. To ensure that the discounted geometric series is finite, the analysis assumes $0 < d(1 + g) < 1$.

Under cooperation, Firm 1 has market share x in a single market, where $0 < x < 1$. In a two-market setting, its shares are x_1 and x_2 . Firm 2 receives the residual share in each market. Deviation is publicly observed. The punishment strategy is a permanent reversion to price competition. In the two-market model, a deviation in either market triggers punishment in both markets at the same time. This simultaneous-punishment assumption makes the strategic linkage explicit and corresponds to the pooling mechanism in the benchmark multimarket-contact model.

For the sales model, S denotes aggregate sales revenue in one market under the cooperative price. When punishment begins, aggregate sales revenue becomes $(1 + e)S$, where e is the proportional change caused by the switch to price competition. A positive e means that lower prices and the associated quantity response increase aggregate revenue; a negative e means that aggregate revenue contracts. The main derivations use $-1 < e < 1$. The lower bound ensures positive sales during punishment, while the upper bound produces an interior deterrence condition. If $e \geq 1$, punishment sales are at

least twice cooperative sales, and collusion cannot be sustained for a firm with a share below one under the present payoff normalisation.

Economically, e combines the price reduction and the quantity response. Writing p^c for the cooperative price, c for the punishment price and $Q(\cdot)$ for market demand, $1 + e = c \cdot Q(c) / [p^c \cdot Q(p^c)]$. Hence $e > 0$ if and only if the arc (midpoint) price elasticity of demand between c and p^c exceeds one; e may also capture market expansion induced by lower prices, such as the entry of price-sensitive consumers. In Sections 3.2–3.4, e is treated as an exogenous reduced-form parameter; Section 3.5 derives it from a linear demand system.

The solution method proceeds in four steps. First, the analysis specifies each firm's cooperative payoff, its one-period deviation payoff and the continuation payoff under permanent punishment. Second, the growing continuation stream is written as a convergent geometric series. Third, the present value of cooperation is compared with the present value of deviation. Fourth, the resulting incentive inequality is solved for the discount factor and compared across objectives and market structures. The same sequence is applied to the single-market and linked-market cases, which makes the source of every difference transparent. Conditions are reported for Firm 1; because Firm 2 receives the residual market share, a cooperative outcome requires the analogous condition to hold for Firm 2 as well. Strict inequalities are used to describe incentives for continued cooperation, while equality identifies the boundary at which a firm is indifferent. No numerical optimisation, simulation, calibration or proprietary data are required. The derivations can therefore be replicated directly from the definitions in Table 1 and the payoff expressions that follow.

Table 1 summarises the model's parameters and restrictions.

Table -1: Model Parameters and Restrictions

Symbol	Definition	Restriction
d	Discount factor applied to future profit or sales	$0 < d < 1$
g	Periodic growth rate of the market	$g > -1$
x, x_1, x_2	Firm 1's market share in one or two markets	$0 < x, x_1, x_2 < 1$
π	Aggregate supracompetitive profit in one market	$\pi > 0$
S	Aggregate sales revenue at the cooperative price in one market	$S > 0$
e	Proportional change in aggregate sales after punishment begins	$-1 < e < 1$
$d(1+g)$	Effective discounted-growth factor	$0 < d(1+g) < 1$

3.2 Benchmark Profit-Maximisation Model

Consider first a single market. Cooperation generates aggregate supracompetitive profit π in the current period. Firm 1 receives $x\pi$. If it continues to cooperate, its discounted profit is

$$V_1^{P,C} = x\pi \sum_{t=0}^{\infty} [d(1+g)]^t = \frac{x\pi}{1-d(1+g)}.$$

If Firm 1 deviates, it captures the entire supracompetitive profit π in the deviation period. From the following period, Bertrand punishment drives supracompetitive profit to zero. The deviation payoff is therefore

$$V_1^{P,D} = \pi.$$

Tacit collusion is sustainable for Firm 1 when $V_1^{P,C} > V_1^{P,D}$. Rearranging gives

$$d > \frac{1-x}{1+g}.$$

The threshold falls as x rises because a larger share increases the cooperative stream. It also falls as g rises because growth makes future cooperative profit more valuable relative to the current deviation gain.

Now consider two markets of equal size and profitability. Firm 1 receives $x_1\pi$ in market 1 and $x_2\pi$ in market 2. Continued cooperation yields

$$V_{1,2}^{P,C} = \frac{(x_1 + x_2)\pi}{1-d(1+g)}.$$

Deviation produces 2π in the current period because the firm captures the supracompetitive profit in both markets, followed by zero supracompetitive profit during punishment.

Cooperation is sustainable when

$$d > \frac{1 - \frac{x_1 + x_2}{2}}{1 + g}.$$

Table 2 compares the single-market and two-market benchmark. In the two-market model, the arithmetic mean $(x_1 + x_2)/2$ replaces the single-market share x . Linking markets is helpful when that mean lowers a binding threshold. For example, a firm with a small share in one market may remain cooperative if its share in the other market is sufficiently large. If both markets reproduce the same share and payoff structure, however, the threshold is unchanged.

Table -2: Conditions for Sustaining Tacit Collusion under Profit Maximisation

Competitive setting	Condition for Firm 1
Single market	$d > \frac{1-x}{1+g}$
Two linked markets	$d > \frac{1 - \frac{x_1 + x_2}{2}}{1 + g}$

3.3 Single-Market Sales-Maximisation Model

The sales model retains the competitive assumptions but changes the objective. Firms maximise discounted sales revenue subject to non-negative profit. Under the cooperative price, aggregate market sales are S , and Firm 1 receives xS . Continued cooperation produces

$$V_1^{S,C} = xS \sum_{t=0}^{\infty} [d(1+g)]^t = \frac{xS}{1-d(1+g)}.$$

If Firm 1 deviates, it captures the only market in the deviation period and therefore obtains sales S . From the next period, Firm 2 matches the price. Identical costs and a homogeneous product imply equal shares during punishment, so each firm receives $(1+e)S/2$ per period, adjusted for subsequent market growth. Firm 1's deviation payoff is

$$V_1^{S,D} = S + \frac{(1+e)S}{2} \sum_{t=1}^{\infty} [d(1+g)]^t.$$

Using the geometric-series expression and comparing $V_1^{S,C}$ with $V_1^{S,D}$ gives

$$d > \frac{2(1-x)}{(1-e)(1+g)} = \frac{2}{1-e} \left(\frac{1-x}{1+g} \right).$$

Three elements determine this threshold. A larger cooperative share x reduces the appeal of deviation. A higher growth rate g increases the value of future cooperative sales. A larger e raises sales during

punishment and therefore makes deviation more attractive. The last effect has no counterpart in the benchmark profit model because punishment profit is normalised to zero regardless of the demand response.

The boundary cases clarify the mechanism. As e approaches -1 , the punishment market produces almost no sales and the multiplier $2/(1 - e)$ approaches one; the sales condition converges to the profit condition. At $e = 0$, aggregate market sales are unchanged by the price war, each firm retains half of S during punishment, and the required threshold is twice the benchmark threshold. As e approaches one, punishment becomes ineffective because aggregate sales become very large relative to cooperative sales. When $e \geq 1$, no admissible $x < 1$ can satisfy the original incentive inequality under the assumed payoff structure.

3.4 Two-Market Sales-Maximisation Model

Finally, suppose the firms interact in two equal-sized markets. Aggregate cooperative sales are S in each market. Firm 1 receives $(x_1 + x_2)S$ per period. The present value of continued cooperation is

$$V_{1,2}^{S,C} = \frac{(x_1 + x_2)S}{1 - d(1 + g)}.$$

If Firm 1 deviates in the linked strategy, it captures both markets in the deviation period and obtains $2S$. From the next period, simultaneous punishment gives it one half of each market. Its combined sales are therefore $(1 + e)S$ per period, adjusted for growth. The deviation payoff is

$$V_{1,2}^{S,D} = 2S + (1 + e)S \sum_{t=1}^{\infty} [d(1 + g)]^t.$$

Cooperation is sustainable when

$$d > \frac{2 - (x_1 + x_2)}{(1 - e)(1 + g)} = \frac{2}{1 - e} \left[\frac{1 - \frac{x_1 + x_2}{2}}{1 + g} \right].$$

This condition has the same structure as the single-market sales condition, with the average share across linked markets replacing the share in one market. It also shows why multimarket contact should not be treated as an unqualified cause of collusion. Contact changes the incentive constraint only when linking markets changes the effective share or combines otherwise different constraints. Where markets differ in size, growth or punishment response, the arithmetic mean must be replaced by a weighted expression. The equal-market formulation used here is a transparent baseline that can be replicated and extended.

3.5 Endogenising the Demand Response under Linear Demand

The analysis so far treats e as a reduced-form summary of the demand response. To show how it relates to demand primitives, suppose that market demand is linear, $Q(p) = a - bp$, with $a > bc > 0$, where c is the common constant marginal cost. Under cooperation the firms charge the joint-profit-maximising price, and under punishment Bertrand competition drives the price to c . The cooperative price and aggregate cooperative sales revenue are therefore

$$p^m = \frac{a + bc}{2b}, \quad S = p^m Q(p^m) = \frac{(a + bc)(a - bc)}{4b}.$$

Under linear demand the punishment quantity is exactly twice the cooperative quantity, $Q(c) = a - bc = 2Q(p^m)$, so the punishment revenue $cQ(c)$ gives

$$1 + e = \frac{cQ(c)}{S} = \frac{2c}{p^m} = \frac{4bc}{a + bc}, \quad e = \frac{3bc - a}{a + bc}.$$

Because $0 < bc < a$, this expression satisfies $-1 < e < 1$ for every admissible parameter value. The restriction imposed on e in Section 3.1 is therefore not an additional assumption under linear demand but a consequence of the primitives. Moreover, $e > 0$ if and only if $p^m < 2c$, that is, when the collusive mark-up over marginal cost is below 100 per cent: the price war then lowers price proportionally less than it raises quantity, so aggregate revenue expands. Substituting into the multiplier $2/(1 - e)$ derived in Section 3.3 gives

$$\frac{2}{1 - e} = \frac{p^m}{p^m - c} = \frac{a + bc}{a - bc} = \varepsilon(p^m),$$

where $\varepsilon(p^m) = bp^m/Q(p^m)$ is the absolute price elasticity of demand at the collusive price; equivalently, the multiplier is the inverse of the Lerner index $(p^m - c)/p^m$. Combining this with the conditions in Sections 3.3 and 3.4 yields the following result.

Proposition 1. Under linear demand, the minimum discount factor required to sustain tacit collusion under sales maximisation equals the corresponding profit-maximisation threshold multiplied by the price elasticity of demand at the collusive price, in both the single-market and the linked-market settings:

$$d > \varepsilon(p^m) \frac{1 - x}{1 + g}, \quad d > \varepsilon(p^m) \frac{1 - \frac{x_1 + x_2}{2}}{1 + g}.$$

Because the joint-profit-maximising price lies on the elastic portion of demand, $\varepsilon(p^m) > 1$, which reproduces the general result that the sales threshold exceeds the profit threshold. The proposition adds an interpretation in terms of a measurable quantity: the more elastic demand is at the collusive price—or, equivalently, the lower the collusive mark-up—the weaker the deterrent value of punishment for a sales-maximising firm. Since the threshold must lie below one for cooperation to be feasible, the proposition also implies a minimum collusive mark-up below which sales-oriented collusion cannot be sustained for given shares and growth. The linear specification is illustrative. Other demand systems map primitives into e differently; under constant-elasticity demand, for example, e reaches or exceeds one whenever the elasticity is two or more, in which case sales-oriented collusion cannot be sustained under the present payoff structure (Section 3.1).

4. RESULTS AND DISCUSSION

4.1 Comparison of Objective Functions

Table 3 presents the four conditions derived above. The profit model depends on market share and growth. The sales model multiplies the corresponding profit threshold by $2/(1 - e)$. Because $-1 < e < 1$, this multiplier is greater than one. Holding shares and growth constant, a sales-maximising firm must therefore place more weight on future outcomes to sustain tacit collusion.

Table -3: Comparison of Profit- and Sales-Maximisation Conditions

Objective	Single market	Two linked markets
Profit maximisation	$d > \frac{1 - x}{1 + g}$	$d > \frac{1 - \frac{x_1 + x_2}{2}}{1 + g}$
Sales maximisation	$d > \frac{2(1 - x)}{(1 - e)(1 + g)}$	$d > \frac{2 - (x_1 + x_2)}{(1 - e)(1 + g)}$

The economic reason is straightforward. Profit punishment creates a zero supracompetitive-profit state. Sales punishment does not create a zero-sales state. Even after a price war begins, both firms continue to sell. The stream that remains after deviation reduces the opportunity cost of attacking. When the price reduction expands aggregate sales, the remaining stream becomes still more valuable and the deterrent becomes weaker.

The comparison also clarifies that sales maximisation does not imply irrational pricing or disregard for viability. The objective is sales revenue subject to a feasibility constraint, consistent with the managerial theories reviewed above (Baumol 1959; Marris 1964; Williamson 1964). In the punishment equilibrium, price equals marginal cost and economic profit is zero rather than negative. A manager can therefore prefer the larger sales stream without violating the non-negative-profit requirement. If an organisation required a strictly positive minimum profit, the punishment path would need to be modified, but the central result would remain: the severity of punishment depends on what the decision maker values.

What the model adds. Standard multimarket-contact theory shows that linking markets can facilitate collusion by pooling incentive constraints under profit maximisation (Bernheim and Whinston 1990), while the managerial-incentive literature shows that managerial objectives can affect collusive stability (Spagnolo 2005; Lee and Turner 2024). Building on Sakai (2017), the present framework carries forward the insight that the change in aggregate sales during punishment, e , matters under sales maximisation and scales the corresponding profit threshold by $2/(1 - e)$. The new analytical contribution of the present paper is to derive this scaling factor from demand primitives under linear demand and show that it equals the price elasticity of demand at the collusive price. Accordingly, the paper links the earlier sales-oriented framework to a measurable demand condition and implies that mutual-forbearance predictions depend jointly on market overlap, managerial objectives and the demand environment in which punishment occurs.

4.2 Demand Response, Growth and Product-Market Conditions

The parameters g and e capture different dimensions of the market. The growth rate g scales both cooperative and punishment outcomes over time. It increases the value of future cooperation and therefore makes restraint easier to sustain. The parameter e , by contrast, changes the relative payoff of punishment. It describes what happens to aggregate sales when the price regime changes. Treating them separately prevents market growth from being used as a proxy for price responsiveness. In Sections 3.2–3.4, e is a reduced-form representation of the demand response: it summarises, rather than derives, the effect of the price change on quantity and revenue. Section 3.5 shows how it maps into demand primitives in the linear case.

This distinction produces a product-market interpretation. Suppose a market is expanding because adoption is increasing, but a price reduction would also unlock substantial additional demand. A high g favours cooperation, while a high e undermines it. The two forces work in opposite directions. In a mature market, g may be low, but e can also be negative if a price cut largely

transfers customers between firms and reduces revenue per customer. Low growth alone therefore does not establish that collusion will be unstable. The relevant assessment requires both the intertemporal growth path and the revenue response to price competition.

The logic is also useful for industries with scale-oriented performance indicators. Managers can be evaluated on revenue, subscriber counts, units shipped, store coverage or employment supported. These measures are not identical, but each can preserve value during a price war even when profit margins collapse. The present model uses sales revenue because it is measurable and directly comparable across regimes. Extensions could replace S with quantity, users, installed base or employment and derive a corresponding punishment response.

4.3 What Multimarket Contact Does and Does Not Do

The two-market results preserve the core insight of Bernheim and Whinston (1990): strategic linkage matters through pooled incentives. A firm with $x_1 = 0.2$ and $x_2 = 0.8$ has an average share of 0.5. If each market were considered independently, the low-share market would create a strong incentive to deviate. Simultaneous punishment links that temptation to the loss of a much larger cooperative stream in the high-share market. The second market can therefore relax the first market's constraint.

However, adding an identical second market does not change the threshold. If $x_1 = x_2 = x$, the two-market condition reduces to the single-market condition for either objective. This point is important for empirical interpretation. A positive count of shared markets is not sufficient evidence of stronger mutual forbearance. The distribution of market positions, the credibility and reach of punishment, and the corporate objective all shape the result. This conditional interpretation is consistent with evidence that contact effects vary by strategic similarity, market structure and demand substitution (Ciliberto and Williams 2014; Gimeno and Woo 1996; Li and Greenwood 2004; Yu and Cannella 2013).

The simultaneous-punishment assumption is also substantive. It requires decision makers to observe deviation and to coordinate a response across markets. Large diversified firms may delegate local decisions, which could weaken direct coordination. Feinberg and Larson (2024) show that informational linkage can still support collusion under decentralisation, but their mechanism depends on noisy signals and managerial inference. In a sales-oriented organisation, local targets can create an additional complication: a manager responsible for revenue in one market may welcome the volume generated by punishment even when headquarters prefers group-wide restraint. Organisational design and incentive systems therefore become part of the multimarket-contact mechanism rather than background conditions.

4.4 Managerial and Policy Implications

For managers, the analysis suggests that a cooperative industry structure may be less stable than conventional profit-based reasoning indicates. Firms whose internal targets strongly reward revenue growth, capacity utilisation or employment maintenance may have greater incentives to initiate or continue price competition. A rival's history of thin margins does not necessarily imply that it will retreat if its strategic objective is to preserve

sales. Competitive intelligence should therefore examine performance measures, governance and stakeholder commitments alongside market overlap.

For boards and investors, the model highlights a potential conflict between revenue-oriented managerial incentives and profitability. Linking bonuses, promotion or prestige primarily to scale can weaken the internal cost of aggressive pricing. Even without an explicit intention to disrupt coordination, managers may select strategies that maintain sales and employment while depressing industry profit. Clear constraints on acceptable margins or returns can change the effective objective and hence the repeated-game incentives.

For competition authorities, multimarket contact remains a relevant screening variable, but it should not be interpreted mechanically. Higher overlap may facilitate tacit coordination, as the airline evidence and later research indicate (Ciliberto and Williams 2014; Evans and Kessides 1994). Yet alternative objectives can produce the opposite empirical pattern: extensive contact alongside persistent price rivalry and weak profitability. Authorities should distinguish explicit agreements from tacit outcomes and should consider whether non-price dimensions such as quality or investment carry the effects of mutual forbearance. Wilson's (2025) findings on internet-service quality illustrate why price data alone may be incomplete.

The analysis does not imply that stakeholder orientation is inherently pro-competitive or anti-competitive. A commitment to employment can encourage output and price competition in one setting, while stable relationships or cross-market retaliation can restrain rivalry in another. The result is conditional on how the objective changes the payoff from cooperation and punishment. This conditionality is precisely why an explicit objective function improves the analysis.

4.5 Robustness and Extensions

Not all simplifying assumptions are equally important for interpreting the results. The comparison between objectives depends most directly on how sales evolve during the punishment phase, which the model summarises in two assumptions: aggregate punishment sales equal $(1 + e)S$, and they are divided equally between symmetric firms. Any change that raises the deviator's sales during punishment—a larger e or a punishment-phase share above one half—weakens deterrence for a sales-maximising firm, whereas changes that reduce punishment sales strengthen it. A second key assumption is that the deviator captures the full cooperative sales S in the deviation period; with differentiated products or capacity constraints, deviation revenue would be smaller and the sales threshold correspondingly lower. By contrast, the assumptions of equal-sized markets and simultaneous punishment mainly affect the multimarket results, by determining whether the relevant share is a simple or a weighted average, and do not alter the comparison between objectives. The conclusions concerning sales maximisation should therefore be read as conditional on the behaviour of sales during punishment.

The main comparative result is robust to unequal market sizes in qualitative terms. If market 1 and market 2 have cooperative sales S_1 and S_2 , the relevant share becomes a sales-weighted average rather than $(x_1 + x_2)/2$. Different growth rates and punishment responses would similarly

enter as market-specific discounted streams. Linking markets facilitates restraint when high-value cooperative positions offset low-value positions that would otherwise invite deviation.

The equal-objective assumption is another useful point of departure rather than a necessary description of every industry. If Firm 1 maximises sales while Firm 2 maximises profit, the firms face different incentive constraints even when their costs and market shares are identical. Firm 1 values the sales that survive punishment, whereas Firm 2 focuses on the loss of supracompetitive profit. Tacit collusion requires both constraints to hold, so the more difficult constraint - ordinarily that of the sales-oriented firm under the present assumptions - determines whether the cooperative path is feasible. This observation suggests that heterogeneity in governance or executive incentives can destabilise an otherwise symmetric market. It also implies that a change in ownership or remuneration at one firm can alter industry conduct without a change in concentration, demand or technology. Sakai (2017) derives these mixed-objective conditions formally for the two-market case and shows that collusion is easiest to sustain when the sales-maximising firm holds the larger combined share, and that the range of share configurations in which linkage facilitates collusion in both markets is narrower than when both firms share the same objective.

A strictly positive minimum-profit constraint would place the sales objective between the two polar cases developed here. If the required profit floor is high, the firm cannot accept marginal-cost pricing during punishment and its feasible sales stream becomes smaller. The sales threshold would then move towards the profit threshold. If the floor is low, the present sales model remains a close approximation. This extension would allow future work to represent objectives as a continuum rather than a binary choice. It would also create empirically testable predictions: firms with revenue-based incentives and low margin requirements should respond more aggressively to price reductions than firms with return-based incentives or binding debt covenants.

The assumption of permanent grim-trigger punishment is deliberately strong. Finite punishment would leave a larger continuation value after deviation and generally make collusion harder to sustain. Imperfect monitoring would introduce the possibility of mistaken punishment, while asymmetric costs would change post-deviation shares. These extensions affect the precise threshold but do not remove the conceptual distinction: when decision makers value sales, the amount of sales remaining during punishment must enter the incentive comparison.

Endogenising e beyond the linear case of Section 3.5 is the most important next step. In the present model, e summarises the combined effect of a lower price and the quantity response on aggregate revenue. A demand system could derive e from price elasticity, substitution and market expansion. Such a model would also allow the deviation-period revenue to differ from cooperative sales rather than using the normalisation S . Empirical work could then test whether revenue-oriented firms are less likely to exhibit mutual forbearance in markets where price reductions expand demand.

5. CONCLUSION

This paper examines tacit collusion under multimarket competition when firms maximise sales revenue rather than profit. The standard profit model makes punishment effective by eliminating supracompetitive profit after deviation. A sales-oriented firm, however, continues to obtain sales during the punishment phase. The amount that remains depends on how aggregate market sales respond to the price war. Introducing this response changes the condition for sustaining cooperation.

The first conclusion, carried forward from Sakai (2017), is that market share and market growth remain important under both objectives, but they are not sufficient under sales maximisation. The proportional change in aggregate sales after price competition begins, denoted by e , becomes an additional determinant. A larger e weakens punishment and raises the degree of future orientation required for tacit collusion.

The second conclusion is comparative. For $-1 < e < 1$, the sales-maximisation threshold equals the corresponding profit threshold multiplied by $2/(1 - e)$, which is greater than one. Tacit collusion is therefore more difficult to sustain under the sales objective, holding market shares and growth constant. The new linear-demand extension in this paper shows that this multiplier equals the price elasticity of demand at the collusive price, linking the result to an observable mark-up.

The third conclusion concerns multimarket contact. Linking two markets replaces the single-market share with the average share across the linked markets. Contact can stabilise cooperation when a strong position in one market offsets a weak position in another, but it does not automatically do so when markets are symmetric. Taken together, these results support a conditional view of mutual forbearance in which asymmetry, punishment reach, organisational objectives and demand conditions jointly determine rivalry.

The model has limitations. It assumes two firms, homogeneous products, identical costs, perfect observation, equal-sized markets, simultaneous permanent punishment and an exogenous demand-response parameter. It also represents sales maximisation with a non-negative-profit constraint rather than modelling a richer combination of profit, revenue, employment and managerial utility. These simplifications make the comparison transparent but limit direct prediction for any single industry.

Future research should derive the demand response from explicit price and quantity functions beyond the linear case, allow market-specific growth and punishment, and model heterogeneous objectives across rivals. Empirical tests could combine data on multimarket contact with executive incentives, ownership, employment commitments and revenue sensitivity. Such work would help establish when market overlap produces mutual forbearance and when sales-oriented objectives instead sustain aggressive rivalry.

Data Availability Statement: No empirical dataset was used in this theoretical study. All assumptions and analytical derivations required to reproduce the results are contained in the manuscript.

AI Use Disclosure: The research question, core theoretical model, and the single- and two-market conditions originate in the author's earlier Japanese-

language work (Sakai 2017), which predates current generative AI tools. In preparing and revising the present English-language manuscript, OpenAI ChatGPT and Anthropic Claude were used to (i) translate and draft English text from the author's Japanese work and notes, including rewriting and restructuring; (ii) assist with language editing and journal-template formatting; (iii) assist in identifying and checking relevant literature; and (iv) explore and check algebraic derivations, including the linear-demand extension in Section 3.5. The author reviewed all AI-assisted text and cited sources and takes full responsibility for the final model, derivations, interpretations, and conclusions. All mathematical derivations, including Proposition 1 in Section 3.5, were independently verified by the author.

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